

Corrigé de l'exercice 1Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = -3x^2 + 8x + 8$$

$$= -3 \times \left(x^2 - \frac{8}{3}x - \frac{8}{3} \right)$$

$$= -3 \times \left(\left(x - \frac{4}{3} \right)^2 - \left(\frac{4}{3} \right)^2 + \frac{-8}{3} \right)$$

$$= -3 \times \left(\left(x - \frac{4}{3} \right)^2 + \frac{-16}{9} + \frac{-8 \times 3}{3 \times 3} \right)$$

$$= -3 \times \left(\left(x - \frac{4}{3} \right)^2 + \frac{-16}{9} + \frac{-24}{9} \right)$$

$$= -3 \times \left(\left(x - \frac{4}{3} \right)^2 + \frac{-40}{9} \right)$$

$$= -3 \times \left(x - \frac{4}{3} \right)^2 + \frac{-40 \times 3 \times (-1)}{3 \times 3}$$

$$P(x) = -3 \times \left(x - \frac{4}{3} \right)^2 + \frac{40}{3}$$

$$Q(x) = 25x^2 + 90x + 81$$

$$= (5x + 9)^2$$

$$= \left(5 \times \left(x + \frac{9}{5} \right) \right)^2$$

$$Q(x) = 25 \times \left(x + \frac{9}{5} \right)^2$$

$$S(x) = x^2 - 8x + 9$$

$$= (x - 4)^2 - 4^2 + 9$$

$$= (x - 4)^2 - 16 + 9$$

$$S(x) = (x - 4)^2 - 7$$

$$R(x) = x^2 + 11x + 4$$

$$= \left(x + \frac{11}{2} \right)^2 - \left(\frac{11}{2} \right)^2 + 4$$

$$= \left(x + \frac{11}{2} \right)^2 + \frac{-121}{4} + \frac{4 \times 4}{1 \times 4}$$

$$= \left(x + \frac{11}{2} \right)^2 + \frac{-121}{4} + \frac{16}{4}$$

$$R(x) = \left(x + \frac{11}{2} \right)^2 + \frac{-105}{4}$$

Corrigé de l'exercice 2Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 9x^2 + 36x + 36$$

$$= (3x + 6)^2$$

$$= \left(3 \times \left(x + \frac{6}{3} \right) \right)^2$$

$$P(x) = 9 \times (x + 2)^2$$

$$Q(x) = x^2 + 5x - 9$$

$$= \left(x + \frac{5}{2} \right)^2 - \left(\frac{5}{2} \right)^2 - 9$$

$$= \left(x + \frac{5}{2} \right)^2 + \frac{-25}{4} - \frac{9 \times 4}{1 \times 4}$$

$$= \left(x + \frac{5}{2} \right)^2 + \frac{-25}{4} - \frac{36}{4}$$

$$Q(x) = \left(x + \frac{5}{2} \right)^2 + \frac{-61}{4}$$

$$S(x) = x^2 - 4x + 9$$

$$= (x - 2)^2 - 2^2 + 9$$

$$= (x - 2)^2 - 4 + 9$$

$$S(x) = (x - 2)^2 + 5$$

$$R(x) = 2x^2 - 5x + 3$$

$$= 2 \times \left(x^2 - \frac{5}{2}x + \frac{3}{2} \right)$$

$$= 2 \times \left(\left(x - \frac{5}{4} \right)^2 - \left(\frac{5}{4} \right)^2 + \frac{3}{2} \right)$$

$$= 2 \times \left(\left(x - \frac{5}{4} \right)^2 + \frac{-25}{16} + \frac{3 \times 8}{2 \times 8} \right)$$

$$= 2 \times \left(\left(x - \frac{5}{4} \right)^2 + \frac{-25}{16} + \frac{24}{16} \right)$$

$$= 2 \times \left(\left(x - \frac{5}{4} \right)^2 + \frac{-1}{16} \right)$$

$$= 2 \times \left(x - \frac{5}{4} \right)^2 + \frac{-1 \times 2}{2 \times 8}$$

$$R(x) = 2 \times \left(x - \frac{5}{4} \right)^2 + \frac{-1}{8}$$

Corrigé de l'exercice 3

Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= x^2 - 16x - 2 \\ &= (x - 8)^2 - 8^2 - 2 \\ &= (x - 8)^2 - 64 - 2 \end{aligned}$$

$$\boxed{P(x) = (x - 8)^2 - 66}$$

$$\begin{aligned} Q(x) &= 5x^2 - 6x + 8 \\ &= 5 \times \left(x^2 - \frac{6}{5}x + \frac{8}{5} \right) \\ &= 5 \times \left(\left(x - \frac{3}{5} \right)^2 - \left(\frac{3}{5} \right)^2 + \frac{8}{5} \right) \\ &= 5 \times \left(\left(x - \frac{3}{5} \right)^2 + \frac{-9}{25} + \frac{8 \times 5}{5 \times 5} \right) \end{aligned}$$

$$\begin{aligned} &= 5 \times \left(\left(x - \frac{3}{5} \right)^2 + \frac{-9}{25} + \frac{40}{25} \right) \\ &= 5 \times \left(\left(x - \frac{3}{5} \right)^2 + \frac{31}{25} \right) \\ &= 5 \times \left(x - \frac{3}{5} \right)^2 + \frac{31 \times 5}{5 \times 5} \end{aligned}$$

$$\boxed{Q(x) = 5 \times \left(x - \frac{3}{5} \right)^2 + \frac{31}{5}}$$

$$\begin{aligned} S(x) &= x^2 - 7x + 8 \\ &= \left(x - \frac{7}{2} \right)^2 - \left(\frac{7}{2} \right)^2 + 8 \\ &= \left(x - \frac{7}{2} \right)^2 + \frac{-49}{4} + \frac{8 \times 4}{1 \times 4} \\ &= \left(x - \frac{7}{2} \right)^2 + \frac{-49}{4} + \frac{32}{4} \end{aligned}$$

$$\boxed{S(x) = \left(x - \frac{7}{2} \right)^2 + \frac{-17}{4}}$$

$$\begin{aligned} R(x) &= 81x^2 - 18x + 1 \\ &= (9x - 1)^2 \\ &= \left(9 \times \left(x - \frac{1}{9} \right) \right)^2 \end{aligned}$$

$$\boxed{R(x) = 81 \times \left(x - \frac{1}{9} \right)^2}$$

Corrigé de l'exercice 4

Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= -4x^2 - 8x + 7 \\ &= -4 \times \left(x^2 + 2x - \frac{7}{4} \right) \\ &= -4 \times \left((x + 1)^2 - 1^2 + \frac{-7}{4} \right) \\ &= -4 \times \left((x + 1)^2 - 1 + \frac{-7}{4} \right) \end{aligned}$$

$$\begin{aligned} &= -4 \times \left((x + 1)^2 + \frac{-1 \times 4}{1 \times 4} + \frac{-7}{4} \right) \\ &= -4 \times \left((x + 1)^2 + \frac{-4}{4} + \frac{-7}{4} \right) \\ &= -4 \times \left((x + 1)^2 + \frac{-11}{4} \right) \\ &= -4 \times (x + 1)^2 + \frac{-11 \times 4 \times (-1)}{4 \times 1} \end{aligned}$$

$$\boxed{P(x) = -4 \times (x + 1)^2 + 11}$$

$$\begin{aligned} Q(x) &= x^2 + 7x - 4 \\ &= \left(x + \frac{7}{2} \right)^2 - \left(\frac{7}{2} \right)^2 - 4 \\ &= \left(x + \frac{7}{2} \right)^2 + \frac{-49}{4} - \frac{4 \times 4}{1 \times 4} \\ &= \left(x + \frac{7}{2} \right)^2 + \frac{-49}{4} - \frac{16}{4} \end{aligned}$$

$$\boxed{Q(x) = \left(x + \frac{7}{2} \right)^2 + \frac{-65}{4}}$$

$$\begin{aligned} R(x) &= 4x^2 - 20x + 25 \\ &= (2x - 5)^2 \\ &= \left(2 \times \left(x - \frac{5}{2} \right) \right)^2 \end{aligned}$$

$$\boxed{R(x) = 4 \times \left(x - \frac{5}{2} \right)^2}$$

$$\begin{aligned} S(x) &= x^2 + 10x - 7 \\ &= (x + 5)^2 - 5^2 - 7 \\ &= (x + 5)^2 - 25 - 7 \end{aligned}$$

$$\boxed{S(x) = (x + 5)^2 - 32}$$

Corrigé de l'exercice 5

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 + 3x + 9$$

$$= \left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 9$$

$$= \left(x + \frac{3}{2}\right)^2 + \frac{-9}{4} + \frac{9 \times 4}{1 \times 4}$$

$$= \left(x + \frac{3}{2}\right)^2 + \frac{-9}{4} + \frac{36}{4}$$

$$\boxed{P(x) = \left(x + \frac{3}{2}\right)^2 + \frac{27}{4}}$$

$$Q(x) = 3x^2 - 2x + 9$$

$$= 3 \times \left(x^2 - \frac{2}{3}x + 3\right)$$

$$= 3 \times \left(\left(x - \frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2 + 3\right)$$

$$= 3 \times \left(\left(x - \frac{1}{3}\right)^2 + \frac{-1}{9} + \frac{3 \times 9}{1 \times 9}\right)$$

$$= 3 \times \left(\left(x - \frac{1}{3}\right)^2 + \frac{-1}{9} + \frac{27}{9}\right)$$

$$= 3 \times \left(\left(x - \frac{1}{3}\right)^2 + \frac{26}{9}\right)$$

$$= 3 \times \left(x - \frac{1}{3}\right)^2 + \frac{26 \times 3}{3 \times 3}$$

$$\boxed{Q(x) = 3 \times \left(x - \frac{1}{3}\right)^2 + \frac{26}{3}}$$

$$R(x) = x^2 - 2x - 4$$

$$= (x - 1)^2 - 1^2 - 4$$

$$= (x - 1)^2 - 1 - 4$$

$$\boxed{R(x) = (x - 1)^2 - 5}$$

$$S(x) = 36x^2 - 60x + 25$$

$$= (6x - 5)^2$$

$$= \left(6 \times \left(x - \frac{5}{6}\right)\right)^2$$

$$\boxed{S(x) = 36 \times \left(x - \frac{5}{6}\right)^2}$$

Corrigé de l'exercice 6

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 + 11x + 3$$

$$= \left(x + \frac{11}{2}\right)^2 - \left(\frac{11}{2}\right)^2 + 3$$

$$= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{3 \times 4}{1 \times 4}$$

$$= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{12}{4}$$

$$\boxed{P(x) = \left(x + \frac{11}{2}\right)^2 + \frac{-109}{4}}$$

$$Q(x) = x^2 - 8x + 16$$

$$\boxed{Q(x) = (x - 4)^2}$$

$$R(x) = x^2 - 18x + 3$$

$$= (x - 9)^2 - 9^2 + 3$$

$$= (x - 9)^2 - 81 + 3$$

$$\boxed{R(x) = (x - 9)^2 - 78}$$

$$S(x) = -5x^2 - 6x - 3$$

$$= -5 \times \left(x^2 + \frac{6}{5}x + \frac{3}{5}\right)$$

$$= -5 \times \left(\left(x + \frac{3}{5}\right)^2 - \left(\frac{3}{5}\right)^2 + \frac{3}{5}\right)$$

$$= -5 \times \left(\left(x + \frac{3}{5}\right)^2 + \frac{-9}{25} + \frac{3 \times 5}{5 \times 5}\right)$$

$$= -5 \times \left(\left(x + \frac{3}{5}\right)^2 + \frac{-9}{25} + \frac{15}{25}\right)$$

$$= -5 \times \left(\left(x + \frac{3}{5}\right)^2 + \frac{6}{25}\right)$$

$$= -5 \times \left(x + \frac{3}{5}\right)^2 + \frac{6 \times 5 \times (-1)}{5 \times 5}$$

$$\boxed{S(x) = -5 \times \left(x + \frac{3}{5}\right)^2 + \frac{-6}{5}}$$

Corrigé de l'exercice 7

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 4x^2 - 2x - 5$$

$$= 4 \times \left(x^2 - \frac{1}{2}x - \frac{5}{4} \right)$$

$$= 4 \times \left(\left(x - \frac{1}{4} \right)^2 - \left(\frac{1}{4} \right)^2 + \frac{-5}{4} \right)$$

$$= 4 \times \left(\left(x - \frac{1}{4} \right)^2 + \frac{-1}{16} + \frac{-5 \times 4}{4 \times 4} \right)$$

$$= 4 \times \left(\left(x - \frac{1}{4} \right)^2 + \frac{-1}{16} + \frac{-20}{16} \right)$$

$$= 4 \times \left(\left(x - \frac{1}{4} \right)^2 + \frac{-21}{16} \right)$$

$$= 4 \times \left(x - \frac{1}{4} \right)^2 + \frac{-21 \times 4}{4 \times 4}$$

$$P(x) = 4 \times \left(x - \frac{1}{4} \right)^2 + \frac{-21}{4}$$

$$Q(x) = x^2 - 9x - 8$$

$$= \left(x - \frac{9}{2} \right)^2 - \left(\frac{9}{2} \right)^2 - 8$$

$$= \left(x - \frac{9}{2} \right)^2 + \frac{-81}{4} - \frac{8 \times 4}{1 \times 4}$$

$$= \left(x - \frac{9}{2} \right)^2 + \frac{-81}{4} - \frac{32}{4}$$

$$Q(x) = \left(x - \frac{9}{2} \right)^2 + \frac{-113}{4}$$

$$R(x) = 16x^2 + 24x + 9$$

$$= (4x + 3)^2$$

$$= \left(4 \times \left(x + \frac{3}{4} \right) \right)^2$$

$$R(x) = 16 \times \left(x + \frac{3}{4} \right)^2$$

$$S(x) = x^2 + 18x - 5$$

$$= (x + 9)^2 - 9^2 - 5$$

$$= (x + 9)^2 - 81 - 5$$

$$S(x) = (x + 9)^2 - 86$$

Corrigé de l'exercice 8

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 + 11x - 7$$

$$= \left(x + \frac{11}{2} \right)^2 - \left(\frac{11}{2} \right)^2 - 7$$

$$= \left(x + \frac{11}{2} \right)^2 + \frac{-121}{4} - \frac{7 \times 4}{1 \times 4}$$

$$= \left(x + \frac{11}{2} \right)^2 + \frac{-121}{4} - \frac{28}{4}$$

$$P(x) = \left(x + \frac{11}{2} \right)^2 + \frac{-149}{4}$$

$$Q(x) = 4x^2 - 28x + 49$$

$$= (2x - 7)^2$$

$$= \left(2 \times \left(x - \frac{7}{2} \right) \right)^2$$

$$Q(x) = 4 \times \left(x - \frac{7}{2} \right)^2$$

$$S(x) = x^2 + 16x - 2$$

$$= (x + 8)^2 - 8^2 - 2$$

$$= (x + 8)^2 - 64 - 2$$

$$S(x) = (x + 8)^2 - 66$$

$$R(x) = -2x^2 + 6x + 7$$

$$= -2 \times \left(x^2 - 3x - \frac{7}{2} \right)$$

$$= -2 \times \left(\left(x - \frac{3}{2} \right)^2 - \left(\frac{3}{2} \right)^2 + \frac{-7}{2} \right)$$

$$= -2 \times \left(\left(x - \frac{3}{2} \right)^2 + \frac{-9}{4} + \frac{-7 \times 2}{2 \times 2} \right)$$

$$= -2 \times \left(\left(x - \frac{3}{2} \right)^2 + \frac{-9}{4} + \frac{-14}{4} \right)$$

$$= -2 \times \left(\left(x - \frac{3}{2} \right)^2 + \frac{-23}{4} \right)$$

$$= -2 \times \left(x - \frac{3}{2} \right)^2 + \frac{-23 \times 2 \times (-1)}{2 \times 2}$$

$$R(x) = -2 \times \left(x - \frac{3}{2} \right)^2 + \frac{23}{2}$$