## Corrigé de l'exercice 1

Donner la forme canonique des polynômes $P$ , $Q$ , $R$ et $S$ .

$$P(x) = -4\,x^2 - 6\,x - 2$$
$$= -4 \times \left(x^2 + \frac{3}{2}\,x + \frac{1}{2}\right)$$
$$= -4 \times \left(\left(x + \frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 + \frac{1}{2}\right)$$
$$= -4 \times \left(\left(x + \frac{3}{4}\right)^2 + \frac{-9}{16} + \frac{1_{\times 8}}{2_{\times 8}}\right)$$
$$= -4 \times \left(\left(x + \frac{3}{4}\right)^2 + \frac{-9}{16} + \frac{8}{16}\right)$$
$$= -4 \times \left(\left(x + \frac{3}{4}\right)^2 + \frac{-1}{16}\right)$$
$$= -4 \times \left(x + \frac{3}{4}\right)^2 + \frac{-1 \times \cancel{4} \times (-1) \times 1}{\cancel{4} \times 4}$$
$$\boxed{P(x) = -4 \times \left(x + \frac{3}{4}\right)^2 + \frac{1}{4}}$$

$$Q(x) = 16\,x^2 - 48\,x + 36$$
$$= (4\,x - 6)^2$$
$$= \left(4 \times \left(x - \frac{6}{4}\right)\right)^2$$
$$\boxed{Q(x) = 16 \times \left(x - \frac{3}{2}\right)^2}$$

$$R(x) = x^2 + 10\,x - 6$$
$$= (x + 5)^2 - 5^2 - 6$$
$$= (x + 5)^2 - 25 - 6$$
$$\boxed{R(x) = (x + 5)^2 - 31}$$

$$S(x) = x^2 - 7\,x + 9$$
$$= \left(x - \frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + 9$$
$$= \left(x - \frac{7}{2}\right)^2 + \frac{-49}{4} + \frac{9_{\times 4}}{1_{\times 4}}$$
$$= \left(x - \frac{7}{2}\right)^2 + \frac{-49}{4} + \frac{36}{4}$$
$$\boxed{S(x) = \left(x - \frac{7}{2}\right)^2 + \frac{-13}{4}}$$

## Corrigé de l'exercice 2

Donner la forme canonique des polynômes $P$ , $Q$ , $R$ et $S$ .

$$P(x) = -5\,x^2 + 4\,x + 2$$
$$= -5 \times \left(x^2 - \frac{4}{5}\,x - \frac{2}{5}\right)$$
$$= -5 \times \left(\left(x - \frac{2}{5}\right)^2 - \left(\frac{2}{5}\right)^2 + \frac{-2}{5}\right)$$
$$= -5 \times \left(\left(x - \frac{2}{5}\right)^2 + \frac{-4}{25} + \frac{-2_{\times 5}}{5_{\times 5}}\right)$$
$$= -5 \times \left(\left(x - \frac{2}{5}\right)^2 + \frac{-4}{25} + \frac{-10}{25}\right)$$
$$= -5 \times \left(\left(x - \frac{2}{5}\right)^2 + \frac{-14}{25}\right)$$
$$= -5 \times \left(x - \frac{2}{5}\right)^2 + \frac{-14 \times \cancel{5} \times (-1)}{\cancel{5} \times 5}$$
$$\boxed{P(x) = -5 \times \left(x - \frac{2}{5}\right)^2 + \frac{14}{5}}$$

$$Q(x) = 64\,x^2 + 112\,x + 49$$
$$= (8\,x + 7)^2$$
$$= \left(8 \times \left(x + \frac{7}{8}\right)\right)^2$$
$$\boxed{Q(x) = 64 \times \left(x + \frac{7}{8}\right)^2}$$

$$R(x) = x^2 + 16\,x - 7$$
$$= (x + 8)^2 - 8^2 - 7$$
$$= (x + 8)^2 - 64 - 7$$
$$\boxed{R(x) = (x + 8)^2 - 71}$$

$$S(x) = x^2 - 3\,x - 4$$
$$= \left(x - \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 4$$
$$= \left(x - \frac{3}{2}\right)^2 + \frac{-9}{4} - \frac{4_{\times 4}}{1_{\times 4}}$$
$$= \left(x - \frac{3}{2}\right)^2 + \frac{-9}{4} - \frac{16}{4}$$
$$\boxed{S(x) = \left(x - \frac{3}{2}\right)^2 + \frac{-25}{4}}$$

## Corrigé de l'exercice 3

Donner la forme canonique des polynômes $P$ , $Q$ , $R$ et $S$ .

$$P(x) = x^2 - 3x + 3$$
$$= \left(x - \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 3$$
$$= \left(x - \frac{3}{2}\right)^2 + \frac{-9}{4} + \frac{3 \times 4}{1 \times 4}$$
$$= \left(x - \frac{3}{2}\right)^2 + \frac{-9}{4} + \frac{12}{4}$$
$$\boxed{P(x) = \left(x - \frac{3}{2}\right)^2 + \frac{3}{4}}$$

$$Q(x) = 4x^2 - 5x + 2$$
$$= 4 \times \left(x^2 - \frac{5}{4}x + \frac{1}{2}\right)$$
$$= 4 \times \left(\left(x - \frac{5}{8}\right)^2 - \left(\frac{5}{8}\right)^2 + \frac{1}{2}\right)$$
$$= 4 \times \left(\left(x - \frac{5}{8}\right)^2 + \frac{-25}{64} + \frac{1 \times 32}{2 \times 32}\right)$$
$$= 4 \times \left(\left(x - \frac{5}{8}\right)^2 + \frac{-25}{64} + \frac{32}{64}\right)$$
$$= 4 \times \left(\left(x - \frac{5}{8}\right)^2 + \frac{7}{64}\right)$$
$$= 4 \times \left(x - \frac{5}{8}\right)^2 + \frac{7 \times \cancel{4}}{\cancel{4} \times 16}$$
$$\boxed{Q(x) = 4 \times \left(x - \frac{5}{8}\right)^2 + \frac{7}{16}}$$

$$R(x) = x^2 - 2x - 6$$
$$= (x - 1)^2 - 1^2 - 6$$
$$= (x - 1)^2 - 1 - 6$$
$$\boxed{R(x) = (x - 1)^2 - 7}$$

$$S(x) = 9x^2 - 36x + 36$$
$$= (3x - 6)^2$$
$$= \left(3 \times \left(x - \frac{6}{3}\right)\right)^2$$
$$\boxed{S(x) = 9 \times (x - 2)^2}$$

## Corrigé de l'exercice 4

Donner la forme canonique des polynômes $P$ , $Q$ , $R$ et $S$ .

$$P(x) = 4x^2 - 20x + 25$$
$$= (2x - 5)^2$$
$$= \left(2 \times \left(x - \frac{5}{2}\right)\right)^2$$
$$\boxed{P(x) = 4 \times \left(x - \frac{5}{2}\right)^2}$$

$$R(x) = x^2 - 12x - 6$$
$$= (x - 6)^2 - 6^2 - 6$$
$$= (x - 6)^2 - 36 - 6$$
$$\boxed{R(x) = (x - 6)^2 - 42}$$

$$Q(x) = x^2 + 5x + 1$$
$$= \left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 1$$
$$= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{1 \times 4}{1 \times 4}$$
$$= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{4}{4}$$
$$\boxed{Q(x) = \left(x + \frac{5}{2}\right)^2 + \frac{-21}{4}}$$

$$S(x) = 4x^2 + x - 1$$
$$= 4 \times \left(x^2 + \frac{1}{4}x - \frac{1}{4}\right)$$
$$= 4 \times \left(\left(x + \frac{1}{8}\right)^2 - \left(\frac{1}{8}\right)^2 + \frac{-1}{4}\right)$$
$$= 4 \times \left(\left(x + \frac{1}{8}\right)^2 + \frac{-1}{64} + \frac{-1 \times 16}{4 \times 16}\right)$$
$$= 4 \times \left(\left(x + \frac{1}{8}\right)^2 + \frac{-1}{64} + \frac{-16}{64}\right)$$
$$= 4 \times \left(\left(x + \frac{1}{8}\right)^2 + \frac{-17}{64}\right)$$
$$= 4 \times \left(x + \frac{1}{8}\right)^2 + \frac{-17 \times \cancel{4}}{\cancel{4} \times 16}$$
$$\boxed{S(x) = 4 \times \left(x + \frac{1}{8}\right)^2 + \frac{-17}{16}}$$

## Corrigé de l'exercice 5

Donner la forme canonique des polynômes $P$ , $Q$ , $R$ et $S$ .

$$P(x) = x^2 + 10\,x + 6$$
$$= (x+5)^2 - 5^2 + 6$$
$$= (x+5)^2 - 25 + 6$$
$$\boxed{P(x) = (x+5)^2 - 19}$$

$$Q(x) = 9\,x^2 - 24\,x + 16$$
$$= (3\,x - 4)^2$$
$$= \left(3 \times \left(x - \frac{4}{3}\right)\right)^2$$
$$\boxed{Q(x) = 9 \times \left(x - \frac{4}{3}\right)^2}$$

$$R(x) = -5\,x^2 - 5\,x + 8$$
$$= -5 \times \left(x^2 + x - \frac{8}{5}\right)$$
$$= -5 \times \left(\left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + \frac{-8}{5}\right)$$
$$= -5 \times \left(\left(x + \frac{1}{2}\right)^2 + \frac{-1_{\times 5}}{4_{\times 5}} + \frac{-8_{\times 4}}{5_{\times 4}}\right)$$
$$= -5 \times \left(\left(x + \frac{1}{2}\right)^2 + \frac{-5}{20} + \frac{-32}{20}\right)$$
$$= -5 \times \left(\left(x + \frac{1}{2}\right)^2 + \frac{-37}{20}\right)$$
$$= -5 \times \left(x + \frac{1}{2}\right)^2 + \frac{-37 \times \cancel{5} \times (-1)}{\cancel{5} \times 4}$$
$$\boxed{R(x) = -5 \times \left(x + \frac{1}{2}\right)^2 + \frac{37}{4}}$$

$$S(x) = x^2 + 7\,x - 6$$
$$= \left(x + \frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 - 6$$
$$= \left(x + \frac{7}{2}\right)^2 + \frac{-49}{4} - \frac{6_{\times 4}}{1_{\times 4}}$$
$$= \left(x + \frac{7}{2}\right)^2 + \frac{-49}{4} - \frac{24}{4}$$
$$\boxed{S(x) = \left(x + \frac{7}{2}\right)^2 + \frac{-73}{4}}$$

## Corrigé de l'exercice 6

Donner la forme canonique des polynômes $P$ , $Q$ , $R$ et $S$ .

$$P(x) = x^2 + 8\,x + 16$$
$$\boxed{P(x) = (x+4)^2}$$

$$Q(x) = x^2 + 14\,x - 4$$
$$= (x+7)^2 - 7^2 - 4$$
$$= (x+7)^2 - 49 - 4$$
$$\boxed{Q(x) = (x+7)^2 - 53}$$

$$R(x) = x^2 + 5\,x + 2$$
$$= \left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 2$$
$$= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{2_{\times 4}}{1_{\times 4}}$$
$$= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{8}{4}$$
$$\boxed{R(x) = \left(x + \frac{5}{2}\right)^2 + \frac{-17}{4}}$$

$$S(x) = 3\,x^2 + 4\,x - 5$$
$$= 3 \times \left(x^2 + \frac{4}{3}\,x - \frac{5}{3}\right)$$
$$= 3 \times \left(\left(x + \frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^2 + \frac{-5}{3}\right)$$
$$= 3 \times \left(\left(x + \frac{2}{3}\right)^2 + \frac{-4}{9} + \frac{-5_{\times 3}}{3_{\times 3}}\right)$$
$$= 3 \times \left(\left(x + \frac{2}{3}\right)^2 + \frac{-4}{9} + \frac{-15}{9}\right)$$
$$= 3 \times \left(\left(x + \frac{2}{3}\right)^2 + \frac{-19}{9}\right)$$
$$= 3 \times \left(x + \frac{2}{3}\right)^2 + \frac{-19 \times \cancel{3}}{\cancel{3} \times 3}$$
$$\boxed{S(x) = 3 \times \left(x + \frac{2}{3}\right)^2 + \frac{-19}{3}}$$

## Corrigé de l'exercice 7

Donner la forme canonique des polynômes $P$ , $Q$ , $R$ et $S$ .

$$P(x) = 5\,x^2 - 6\,x + 1$$
$$= 5 \times \left(x^2 - \frac{6}{5}\,x + \frac{1}{5}\right)$$
$$= 5 \times \left(\left(x - \frac{3}{5}\right)^2 - \left(\frac{3}{5}\right)^2 + \frac{1}{5}\right)$$
$$= 5 \times \left(\left(x - \frac{3}{5}\right)^2 + \frac{-9}{25} + \frac{1_{\times 5}}{5_{\times 5}}\right)$$
$$= 5 \times \left(\left(x - \frac{3}{5}\right)^2 + \frac{-9}{25} + \frac{5}{25}\right)$$
$$= 5 \times \left(\left(x - \frac{3}{5}\right)^2 + \frac{-4}{25}\right)$$
$$= 5 \times \left(x - \frac{3}{5}\right)^2 + \frac{-4 \times \not{5}}{\not{5} \times 5}$$
$$\boxed{P(x) = 5 \times \left(x - \frac{3}{5}\right)^2 + \frac{-4}{5}}$$

$$Q(x) = x^2 + 5\,x + 5$$
$$= \left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 5$$
$$= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{5_{\times 4}}{1_{\times 4}}$$
$$= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} + \frac{20}{4}$$
$$\boxed{Q(x) = \left(x + \frac{5}{2}\right)^2 + \frac{-5}{4}}$$

$$R(x) = 49\,x^2 + 112\,x + 64$$
$$= (7\,x + 8)^2$$
$$= \left(7 \times \left(x + \frac{8}{7}\right)\right)^2$$
$$\boxed{R(x) = 49 \times \left(x + \frac{8}{7}\right)^2}$$

$$S(x) = x^2 + 16\,x - 2$$
$$= (x + 8)^2 - 8^2 - 2$$
$$= (x + 8)^2 - 64 - 2$$
$$\boxed{S(x) = (x + 8)^2 - 66}$$

## Corrigé de l'exercice 8

Donner la forme canonique des polynômes $P$ , $Q$ , $R$ et $S$ .

$$P(x) = 25\,x^2 + 40\,x + 16$$
$$= (5\,x + 4)^2$$
$$= \left(5 \times \left(x + \frac{4}{5}\right)\right)^2$$
$$\boxed{P(x) = 25 \times \left(x + \frac{4}{5}\right)^2}$$

$$Q(x) = x^2 + 16\,x + 9$$
$$= (x + 8)^2 - 8^2 + 9$$
$$= (x + 8)^2 - 64 + 9$$
$$\boxed{Q(x) = (x + 8)^2 - 55}$$

$$R(x) = -5\,x^2 - 5\,x + 2$$
$$= -5 \times \left(x^2 + x - \frac{2}{5}\right)$$
$$= -5 \times \left(\left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + \frac{-2}{5}\right)$$
$$= -5 \times \left(\left(x + \frac{1}{2}\right)^2 + \frac{-1_{\times 5}}{4_{\times 5}} + \frac{-2_{\times 4}}{5_{\times 4}}\right)$$
$$= -5 \times \left(\left(x + \frac{1}{2}\right)^2 + \frac{-5}{20} + \frac{-8}{20}\right)$$
$$= -5 \times \left(\left(x + \frac{1}{2}\right)^2 + \frac{-13}{20}\right)$$
$$= -5 \times \left(x + \frac{1}{2}\right)^2 + \frac{-13 \times \not{5} \times (-1)}{\not{5} \times 4}$$
$$\boxed{R(x) = -5 \times \left(x + \frac{1}{2}\right)^2 + \frac{13}{4}}$$

$$S(x) = x^2 - 7\,x - 6$$
$$= \left(x - \frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 - 6$$
$$= \left(x - \frac{7}{2}\right)^2 + \frac{-49}{4} - \frac{6_{\times 4}}{1_{\times 4}}$$
$$= \left(x - \frac{7}{2}\right)^2 + \frac{-49}{4} - \frac{24}{4}$$
$$\boxed{S(x) = \left(x - \frac{7}{2}\right)^2 + \frac{-73}{4}}$$