

Corrigé de l'exercice 1Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= x^2 + 14x - 9 \\ &= (x + 7)^2 - 7^2 - 9 \\ &= (x + 7)^2 - 49 - 9 \end{aligned}$$

$$P(x) = (x + 7)^2 - 58$$

$$\begin{aligned} S(x) &= 49x^2 + 42x + 9 \\ &= (7x + 3)^2 \\ &= \left(7 \times \left(x + \frac{3}{7}\right)\right)^2 \end{aligned}$$

$$S(x) = 49 \times \left(x + \frac{3}{7}\right)^2$$

$$\begin{aligned} Q(x) &= x^2 + 5x - 4 \\ &= \left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 - 4 \\ &= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} - \frac{4 \times 4}{1 \times 4} \\ &= \left(x + \frac{5}{2}\right)^2 + \frac{-25}{4} - \frac{16}{4} \end{aligned}$$

$$Q(x) = \left(x + \frac{5}{2}\right)^2 + \frac{-41}{4}$$

$$\begin{aligned} R(x) &= -2x^2 + 7x - 6 \\ &= -2 \times \left(x^2 - \frac{7}{2}x + 3\right) \\ &= -2 \times \left(\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + 3\right) \\ &= -2 \times \left(\left(x - \frac{7}{4}\right)^2 + \frac{-49}{16} + \frac{3 \times 16}{1 \times 16}\right) \\ &= -2 \times \left(\left(x - \frac{7}{4}\right)^2 + \frac{-49}{16} + \frac{48}{16}\right) \\ &= -2 \times \left(\left(x - \frac{7}{4}\right)^2 + \frac{-1}{16}\right) \\ &= -2 \times \left(x - \frac{7}{4}\right)^2 + \frac{-1 \times 2 \times (-1) \times 1}{2 \times 8} \end{aligned}$$

$$R(x) = -2 \times \left(x - \frac{7}{4}\right)^2 + \frac{1}{8}$$

Corrigé de l'exercice 2Donner la forme canonique des polynômes P , Q , R et S .

$$\begin{aligned} P(x) &= 5x^2 - 4x - 6 \\ &= 5 \times \left(x^2 - \frac{4}{5}x - \frac{6}{5}\right) \\ &= 5 \times \left(\left(x - \frac{2}{5}\right)^2 - \left(\frac{2}{5}\right)^2 + \frac{-6}{5}\right) \\ &= 5 \times \left(\left(x - \frac{2}{5}\right)^2 + \frac{-4}{25} + \frac{-6 \times 5}{5 \times 5}\right) \\ &= 5 \times \left(\left(x - \frac{2}{5}\right)^2 + \frac{-4}{25} + \frac{-30}{25}\right) \\ &= 5 \times \left(\left(x - \frac{2}{5}\right)^2 + \frac{-34}{25}\right) \\ &= 5 \times \left(x - \frac{2}{5}\right)^2 + \frac{-34 \times 5}{5 \times 5} \end{aligned}$$

$$P(x) = 5 \times \left(x - \frac{2}{5}\right)^2 + \frac{-34}{5}$$

$$Q(x) = x^2 - 12x + 36$$

$$Q(x) = (x - 6)^2$$

$$S(x) = x^2 + 14x - 2$$

$$= (x + 7)^2 - 7^2 - 2$$

$$= (x + 7)^2 - 49 - 2$$

$$S(x) = (x + 7)^2 - 51$$

$$R(x) = x^2 + 9x + 5$$

$$= \left(x + \frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 + 5$$

$$= \left(x + \frac{9}{2}\right)^2 + \frac{-81}{4} + \frac{5 \times 4}{1 \times 4}$$

$$= \left(x + \frac{9}{2}\right)^2 + \frac{-81}{4} + \frac{20}{4}$$

$$R(x) = \left(x + \frac{9}{2}\right)^2 + \frac{-61}{4}$$

Corrigé de l'exercice 3

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 16x^2 + 8x + 1$$

$$= (4x + 1)^2$$

$$= \left(4 \times \left(x + \frac{1}{4}\right)\right)^2$$

$$\boxed{P(x) = 16 \times \left(x + \frac{1}{4}\right)^2}$$

$$R(x) = x^2 + 12x + 9$$

$$= (x + 6)^2 - 6^2 + 9$$

$$= (x + 6)^2 - 36 + 9$$

$$\boxed{R(x) = (x + 6)^2 - 27}$$

$$Q(x) = x^2 - 7x + 7$$

$$= \left(x - \frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + 7$$

$$= \left(x - \frac{7}{2}\right)^2 + \frac{-49}{4} + \frac{7 \times 4}{1 \times 4}$$

$$= \left(x - \frac{7}{2}\right)^2 + \frac{-49}{4} + \frac{28}{4}$$

$$\boxed{Q(x) = \left(x - \frac{7}{2}\right)^2 + \frac{-21}{4}}$$

$$S(x) = -3x^2 + 8x - 3$$

$$= -3 \times \left(x^2 - \frac{8}{3}x + 1\right)$$

$$= -3 \times \left(\left(x - \frac{4}{3}\right)^2 - \left(\frac{4}{3}\right)^2 + 1\right)$$

$$= -3 \times \left(\left(x - \frac{4}{3}\right)^2 + \frac{-16}{9} + \frac{1 \times 9}{1 \times 9}\right)$$

$$= -3 \times \left(\left(x - \frac{4}{3}\right)^2 + \frac{-16}{9} + \frac{9}{9}\right)$$

$$= -3 \times \left(\left(x - \frac{4}{3}\right)^2 + \frac{-7}{9}\right)$$

$$= -3 \times \left(x - \frac{4}{3}\right)^2 + \frac{-7 \times 3 \times (-1)}{3 \times 3}$$

$$\boxed{S(x) = -3 \times \left(x - \frac{4}{3}\right)^2 + \frac{7}{3}}$$

Corrigé de l'exercice 4

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 25x^2 + 50x + 25$$

$$= (5x + 5)^2$$

$$= \left(5 \times \left(x + \frac{5}{5}\right)\right)^2$$

$$\boxed{P(x) = 25 \times (x + 1)^2}$$

$$R(x) = x^2 - 10x - 7$$

$$= (x - 5)^2 - 5^2 - 7$$

$$= (x - 5)^2 - 25 - 7$$

$$\boxed{R(x) = (x - 5)^2 - 32}$$

$$Q(x) = 2x^2 - x + 2$$

$$= 2 \times \left(x^2 - \frac{1}{2}x + 1\right)$$

$$= 2 \times \left(\left(x - \frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 + 1\right)$$

$$= 2 \times \left(\left(x - \frac{1}{4}\right)^2 + \frac{-1}{16} + \frac{1 \times 16}{1 \times 16}\right)$$

$$= 2 \times \left(\left(x - \frac{1}{4}\right)^2 + \frac{-1}{16} + \frac{16}{16}\right)$$

$$= 2 \times \left(\left(x - \frac{1}{4}\right)^2 + \frac{15}{16}\right)$$

$$= 2 \times \left(x - \frac{1}{4}\right)^2 + \frac{15 \times 2}{2 \times 8}$$

$$\boxed{Q(x) = 2 \times \left(x - \frac{1}{4}\right)^2 + \frac{15}{8}}$$

$$S(x) = x^2 - 9x - 1$$

$$= \left(x - \frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 - 1$$

$$= \left(x - \frac{9}{2}\right)^2 + \frac{-81}{4} - \frac{1 \times 4}{1 \times 4}$$

$$= \left(x - \frac{9}{2}\right)^2 + \frac{-81}{4} - \frac{4}{4}$$

$$\boxed{S(x) = \left(x - \frac{9}{2}\right)^2 + \frac{-85}{4}}$$

Corrigé de l'exercice 5

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 3x^2 - 9x + 4$$

$$= 3 \times \left(x^2 - 3x + \frac{4}{3} \right)$$

$$= 3 \times \left(\left(x - \frac{3}{2} \right)^2 - \left(\frac{3}{2} \right)^2 + \frac{4}{3} \right)$$

$$= 3 \times \left(\left(x - \frac{3}{2} \right)^2 + \frac{-9 \times 3}{4 \times 3} + \frac{4 \times 4}{3 \times 4} \right)$$

$$= 3 \times \left(\left(x - \frac{3}{2} \right)^2 + \frac{-27}{12} + \frac{16}{12} \right)$$

$$= 3 \times \left(\left(x - \frac{3}{2} \right)^2 + \frac{-11}{12} \right)$$

$$= 3 \times \left(x - \frac{3}{2} \right)^2 + \frac{-11 \times 3}{3 \times 4}$$

$$P(x) = 3 \times \left(x - \frac{3}{2} \right)^2 + \frac{-11}{4}$$

$$Q(x) = x^2 + 11x - 3$$

$$= \left(x + \frac{11}{2} \right)^2 - \left(\frac{11}{2} \right)^2 - 3$$

$$= \left(x + \frac{11}{2} \right)^2 + \frac{-121}{4} - \frac{3 \times 4}{1 \times 4}$$

$$= \left(x + \frac{11}{2} \right)^2 + \frac{-121}{4} - \frac{12}{4}$$

$$Q(x) = \left(x + \frac{11}{2} \right)^2 + \frac{-133}{4}$$

$$R(x) = 9x^2 - 30x + 25$$

$$= (3x - 5)^2$$

$$= \left(3 \times \left(x - \frac{5}{3} \right) \right)^2$$

$$R(x) = 9 \times \left(x - \frac{5}{3} \right)^2$$

$$S(x) = x^2 - 18x - 8$$

$$= (x - 9)^2 - 9^2 - 8$$

$$= (x - 9)^2 - 81 - 8$$

$$S(x) = (x - 9)^2 - 89$$

Corrigé de l'exercice 6

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 5x^2 + 8x - 5$$

$$= 5 \times \left(x^2 + \frac{8}{5}x - 1 \right)$$

$$= 5 \times \left(\left(x + \frac{4}{5} \right)^2 - \left(\frac{4}{5} \right)^2 - 1 \right)$$

$$= 5 \times \left(\left(x + \frac{4}{5} \right)^2 + \frac{-16}{25} - \frac{1 \times 25}{1 \times 25} \right)$$

$$= 5 \times \left(\left(x + \frac{4}{5} \right)^2 + \frac{-16}{25} - \frac{25}{25} \right)$$

$$= 5 \times \left(\left(x + \frac{4}{5} \right)^2 + \frac{-41}{25} \right)$$

$$= 5 \times \left(x + \frac{4}{5} \right)^2 + \frac{-41 \times 5}{5 \times 5}$$

$$P(x) = 5 \times \left(x + \frac{4}{5} \right)^2 + \frac{-41}{5}$$

$$Q(x) = x^2 - 11x - 2$$

$$= \left(x - \frac{11}{2} \right)^2 - \left(\frac{11}{2} \right)^2 - 2$$

$$= \left(x - \frac{11}{2} \right)^2 + \frac{-121}{4} - \frac{2 \times 4}{1 \times 4}$$

$$= \left(x - \frac{11}{2} \right)^2 + \frac{-121}{4} - \frac{8}{4}$$

$$Q(x) = \left(x - \frac{11}{2} \right)^2 + \frac{-129}{4}$$

$$R(x) = x^2 + 4x - 9$$

$$= (x + 2)^2 - 2^2 - 9$$

$$= (x + 2)^2 - 4 - 9$$

$$R(x) = (x + 2)^2 - 13$$

$$S(x) = x^2 - 16x + 64$$

$$S(x) = (x - 8)^2$$

Corrigé de l'exercice 7

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = x^2 + 11x + 4$$

$$= \left(x + \frac{11}{2}\right)^2 - \left(\frac{11}{2}\right)^2 + 4$$

$$= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{4 \times 4}{1 \times 4}$$

$$= \left(x + \frac{11}{2}\right)^2 + \frac{-121}{4} + \frac{16}{4}$$

$$P(x) = \left(x + \frac{11}{2}\right)^2 + \frac{-105}{4}$$

$$Q(x) = 16x^2 + 56x + 49$$

$$= (4x + 7)^2$$

$$= \left(4 \times \left(x + \frac{7}{4}\right)\right)^2$$

$$Q(x) = 16 \times \left(x + \frac{7}{4}\right)^2$$

$$R(x) = x^2 + 4x - 1$$

$$= (x + 2)^2 - 2^2 - 1$$

$$= (x + 2)^2 - 4 - 1$$

$$R(x) = (x + 2)^2 - 5$$

$$S(x) = -4x^2 - 2x + 2$$

$$= -4 \times \left(x^2 + \frac{1}{2}x - \frac{1}{2}\right)$$

$$= -4 \times \left(\left(x + \frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 + \frac{-1}{2}\right)$$

$$= -4 \times \left(\left(x + \frac{1}{4}\right)^2 + \frac{-1}{16} + \frac{-1 \times 8}{2 \times 8}\right)$$

$$= -4 \times \left(\left(x + \frac{1}{4}\right)^2 + \frac{-1}{16} + \frac{-8}{16}\right)$$

$$= -4 \times \left(\left(x + \frac{1}{4}\right)^2 + \frac{-9}{16}\right)$$

$$= -4 \times \left(x + \frac{1}{4}\right)^2 + \frac{-9 \times 4 \times (-1)}{4 \times 4}$$

$$S(x) = -4 \times \left(x + \frac{1}{4}\right)^2 + \frac{9}{4}$$

Corrigé de l'exercice 8

Donner la forme canonique des polynômes P , Q , R et S .

$$P(x) = 49x^2 + 84x + 36$$

$$= (7x + 6)^2$$

$$= \left(7 \times \left(x + \frac{6}{7}\right)\right)^2$$

$$P(x) = 49 \times \left(x + \frac{6}{7}\right)^2$$

$$R(x) = x^2 + 3x + 1$$

$$= \left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 1$$

$$= \left(x + \frac{3}{2}\right)^2 + \frac{-9}{4} + \frac{1 \times 4}{1 \times 4}$$

$$= \left(x + \frac{3}{2}\right)^2 + \frac{-9}{4} + \frac{4}{4}$$

$$R(x) = \left(x + \frac{3}{2}\right)^2 + \frac{-5}{4}$$

$$Q(x) = x^2 - 2x + 9$$

$$= (x - 1)^2 - 1^2 + 9$$

$$= (x - 1)^2 - 1 + 9$$

$$Q(x) = (x - 1)^2 + 8$$

$$S(x) = -5x^2 + 5x - 8$$

$$= -5 \times \left(x^2 - x + \frac{8}{5}\right)$$

$$= -5 \times \left(\left(x - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + \frac{8}{5}\right)$$

$$= -5 \times \left(\left(x - \frac{1}{2}\right)^2 + \frac{-1 \times 5}{4 \times 5} + \frac{8 \times 4}{5 \times 4}\right)$$

$$= -5 \times \left(\left(x - \frac{1}{2}\right)^2 + \frac{-5}{20} + \frac{32}{20}\right)$$

$$= -5 \times \left(\left(x - \frac{1}{2}\right)^2 + \frac{27}{20}\right)$$

$$= -5 \times \left(x - \frac{1}{2}\right)^2 + \frac{27 \times 5 \times (-1)}{5 \times 4}$$

$$S(x) = -5 \times \left(x - \frac{1}{2}\right)^2 + \frac{-27}{4}$$